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Outline of a Classical Theory of Quantum Physics and Gravitation

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Abstract

It is argued that in the manner in which the Galilean-Newtonian physics may be said to have explained the Ptolemaic-Copernican theories in terms which have since been called classical, so also Milner's theories of the structure of matter may be said to explain present day quantum and relativistic theory. In both cases the former employ the concept of force and the latter, by contrast, are geometrical theories. Milner envisaged space as being stressed, whereas Einstein thought of it as strained. Development of Milner's theory from criticisms and suggestions made by Kilmister has taken it further into the realms of quantum and gravitational physics, where it is found to give a more physically comprehensible explanation of the phenomena. Further, it shows why present day quantum theory is cast in a statistical form. The theory is supported by many predictions such as the ratio of Planck's constant to the mass of the electron, the value of the fine structure constant and reason for apparent variations in past measurements, the magnetic moment of the electron and proton of the stable particles such as the neutron Λ and Σ together with the kaon, and a relation between the universal gravitational constant and Hubble's constant—all within published experimental accuracy. The latest results to be accounted for by the theory are the masses of the newly discovered ψ particles and confirmation of the value of the decay of Newton's gravitational constant obtained from lunar measurements. While this paper is being typed, new particles are rapidly being discovered—the latest being a neutral ψ particle. A short Appendix discusses the significance of these.

1. *Introduction*

The research described in this paper began with an attempt to explain the jump phenomena of quantum physics in terms of the behavior of oscillatory mechanisms or circuits with nonlinear restoring forces (McLachlan, 1950). The need to construct a model of the hydrogen atom led eventually to the discovery of Milner's theories. From then onwards it became increasingly necessary to pay attention to what Berkson (1974) calls the metaphysical standpoints of Milner, Einstein, and the Copenhagen School, in order to disentangle their derivatives and avoid attributing consequences to the wrong logical bases. Such

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a program led to a view of the history of ideas culminating in the present theory which is set out below. This history is in no sense intended to be an account of what actually happened—I am no historian and have not attempted to acquaint myself with the facts of history in time and place, but the account is my subjective view of the progression of ideas ignoring all but the mainstream of this progress. My attitude can be well illustrated by an example from Berkson's book (pp. 338–9). He quotes J. D. Jackson (1962) as saying "It required the genius of J. C. Maxwell · · · to see the inconsistency in the equations and to modify them into a consistent set · · ·." Berkson then comments "Jackson's statement is very far from the truth." In the context in which Berkson is speaking he means historical truth and, from Berkson's account of Maxwell's work, one cannot quarrel with his criticism. Nevertheless as far as the flow of ideas is concerned, Jackson may be said to be right: That is in effect what Maxwell accomplished, however devious the path by which he arrived at the result.

Apart from the attempt to place the new theory in the historical context of ideas, it seems necessary also to state the philosophical or metaphysical (in Berkson's meaning) standpoint. This is closely akin to Piaget (1970) and might be described in Elsasser's (1973) words as "naive realism." Thus from an initial rather vague assumption that nonlinear restoring forces might lead to a classical formulation of quantum phenomena one finds that not only are the values of $\hbar/m_e$ and the fine structure constant calculable to a high degree of accuracy, but also diverging experimental results may be explained. At this stage the model of the hydrogen atom used in the calculations was based rather tenuously upon the ideas of plasma physics. Then Milner's theory allowed more soundly based models, and the two together show how the masses of some of the fundamental particles and their magnetic moments may be calculated. But then it became necessary to reexamine Milner's theory, for he introduced new forces and those obviously must contribute to the mass. Besides all this we now have nothing but electromagnetic fields, albeit with two new ones, and the theory allows for nothing extra to account for gravitation. Thus one is inexorably forced back into the metaphysical speculation about the nature of space and the role of Milner's constant.

In order to be completely sure of the ground, the theory in this paper ought to be rewritten from the basic assumptions, which seem to be the invariance of charge, the conservation of energy, and that all matter may be represented by the electric and magnetic quaternions. At this stage in the development of the theory, however, this would seem to be unnecessary, and the much more urgent task is to show in what manner it underlies present day quantum and relativity theory and where it can provide new insights and results. This is what I have attempted in the following pages.

2. *The Philosophical Standpoint*

Before putting the ideas in this paper into their historical context, something must be said about their relationship to modern quantum theory; for it would seem from much of the writing about the latter that many would have us believe that it will ultimately be found to be based upon fundamental philosophical or, in Berkson's phraseology, metaphysical concepts, although these are as yet imperfectly understood.

Post (1971) contends that a new theory must necessarily subsume the old; Popper (1972) goes further and claims that in a sense it must also contradict the old. If we accept their propositions, it clearly becomes necessary to show how the new theory offers explanations of phenomena hitherto partially or wholly unexplained and also to show where the new and the old ideas conflict. The new theory being classical, it is immediately obvious that it must deny or subsume the uncertainty principle and complementarity. The claim is that it subsumes them.

Before seeking to establish this claim, however, it is necessary to show the implications of the adoption of such principles and the way in which such implications may be refuted. For by evoking them physicists have, willy-nilly, donned the mantle of those who follow Kant in asserting that philosophical knowledge consists in determining the limits of all knowledge (Piaget, 1971, p. 3). Piaget rejects such an approach on the grounds that such theories may be, and frequently are, subsequently contradicted by experiment; but they may also become irrelevant. A good illustration of how this can happen is contained in Russell's (1946) criticism of Bergson, where he uses Zeno's argument of the arrow:

> 'Zeno argues that, since the arrow in its flight at each moment is simply where it is, therefore the arrow in its flight is always at rest. · · · Zeno assumes, tacitly, the essence of the Bergsonian theory of change. That is to say, he assumes that when a thing is in a process of continuous change, even if it is only a change of position, there must be in the thing some internal state of change. He then points out that at each instant the arrow is simply where it is, just as it would be if it were at rest.'

What would have been Zeno's argument if he had known that, according to relativity theory, the observed mass and dimensions of the arrow had been altered by its motion? Such a question may well give rise to further philosophical speculation, but it is scarcely relevant to a mathematical discussion of the motion of the arrow, or any other practical problem.

In like manner it is now claimed that the new theory explains away two of the phenomena of physics that were said to be insurmountable obstacles to the classical theory (Scheibe, 1973, p. 14) and became the metaphysical pillars of quantum theory, and in so doing-subsumes it into a new classical theory. These two phenomena are the quantization of spectra and the dualistic wave–particle behavior of radiation. The manner of solving the first has been briefly outlined above, the second is resolved in terms of a classical model of the photon.

Consider the model of an H atom with an extended electron cloud surrounding a proton, which for present purposes may be considered as a point. In the ground state the electron cloud will be spherical, but when excited it will adopt the well-known configurations, hitherto known as the probability density patterns (White, 1931). Now these patterns are the same as the radiation patterns of a linear oscillating dipole (Stratton, 1941, pp. 438 et seq.), i.e., they are also the radiation patterns of a dipole consisting of two end-on conductors, each one quarter of the fundamental wavelength long. These conductors are carrying standing waves, while the radiation patterns move outwards at the speed of light. By an obvious inversion of the formulas we may clearly have the radiation patterns as stationary standing waves and the waves on the conductors moving away on the axis of the dipole at the speed of light; only this time the radiation patterns are formed from the conducting cloud of the electron for which the axially moving traveling waves are the radiation. These waves will be limited temporally by the quantized movements of the electron cloud and hence will be of finite length, while confinement about the axis of movement is brought about by the structure of the radiating antenna, namely the electron cloud. Thus the particle-wave-like nature of the photon is explicable in classical terms.

With a classical photon, the role of statistics in the behavior of radiation at once becomes apparent; but their role in particle physics is also demonstrated in the paper. Briefly it is shown that the de Broglie, Schrödinger, and Klein-Gordon equations are the phase velocity counterparts of the Milner (group velocity) wave equation. Thus the particle may be represented by one Milner wave equation, but the sum of many phase velocity equations. As each of the latter may be shown to represent an element of the particle, the particle representation of these waves consists of a statistical ensemble.

To some, the return to classical physics, means a return to a deterministic point of view in Bohr's sense of these words as described by Scheibe (1973). According to Scheibe, Bohr assumed that determinism and causality are synonymous. This also appears to be the case in the discussion of Heisenberg's lecture (1961) by Albert Picot, where he says "the uncertainty principle · · · casts doubt on the general theory of causality and determinism"; similarly in the Born–Einstein letters with their discussion on determinism; and Heitler's *Man and Science* (1963), particularly where he says "Newton is the true discoverer of laws that are differential, causal, and deterministic · · · For every change of speed there is a compelling cause, force."

The present theory is certainly causal, but as Rene Thom (1969) has shown, this need not result in determinism: In applying the notion of structural stability to biological processes, Thom was led to the discovery "that a deterministic system may exhibit, in a "structurally stable way," a complete indeterminacy in the qualitative prediction of the final outcome of its evolution. (Tossing a die is a familiar case of such a situation)." As with Zeno's arrow, mathematics and physics may have given a totally unexpected solution to the philosophic problem. To quote again from Elsasser (1973)

> Views of nature other than naive realism have rarely been more than passing fads. They are chosen as counsels of despair when physicists seem unable to find prompt answers to urgent questions, · · · soon

> after the discovery of the formalism of quantum mechanics, the metaphysical subject-object dualism was imposed upon physics in the wake of the Copenhagen school. But for some years now, these ideas have been losing their attractiveness.

Finally, since the next section deals, albeit cursorily, with the history of ideas relevant to the theme of this paper, it is perhaps prudent to state explicitly that my position is completely at variance with that of Collingwood, Kuhn and Toulmin as set out in Toulmin's "Conceptual Revolutions in Science" (1967) insofar as it is relevant to physics and mathematics. Toulmin (loc. cit.) concludes "we · · · must pursue the philosophical analysis of intellectual judgements in the context of a wider, historico-sociological analysis of intellectual traditions generally." Of course these considerations are relevent to enquiries about how new laws of nature are discovered; but the truth proclaimed by the exact sciences is only relative in degree—it does not admit of equal competition.

3. *Historical Development*

The interpretation, as distinct from the literal historical viewpoint, that I wish to put forward follows that of Russell (1946, p. 239):

> Greek astronomy was geometrical, not dynamic. The ancients thought of the motions of the heavenly bodies as uniform and circular, or compounded of circular motions. They had not the concept of force. —With Newton and gravitation a new point of view, less geometrical, was introduced. It is curious to observe that there is a reversion to the geometrical point of view in Einstein's General Theory of Relativity, from which the conception of force, in the Newtonian sense, has been banished.

We may, I think, fairly extend this sequence to assert that Bohr and Heisenberg bear a relation to Einstein similar to that of Ptolemy and Copernicus to Euclid. It is here, however, that the tidy sequence breaks down, for the relativity revolution by-passed Faraday, who was seeking a total field theory in which particles would only be constructs from the field and space would be filled with fields of force. Faraday, and after him Maxwell, failed to produce such a complete theory because, as we now know, they lacked a binding force for the particles which were to be discovered after their time, and it was not until the 1950's that Milner, although largely unnoticed even till today, provided the necessary forces to allow the program to be completed.

For those who cannot resist the temptation to look for a scapegoat for the failure earlier to notice Milner's work, it should be pointed out that Milner failed to quantize his theory, which made it look old-fashioned and out-of-date. Further, his theory did not seem to offer any easier explanation or forecast of the phenomena of particle physics. The present paper seeks to remedy these defects.

Jackson's (1962) view of Maxwell's displacement current has already been referred to in the Introduction. There is a strong parallel in Milner's scalar forces. Milner factorized matter density into electric and magnetic four-vectors, using a biquaternionic algebra (Synge, 1972), and found that the time components of the vectors he had introduced are "just those which are required to make an extended classical electron stable and relativistic" (Gabor, 1960). Gabor goes on to say

> Thus Milner has at last succeeded in fulfilling H. A. Lorentz's programme of a self-consistent electro-dynamics extending into the inside of the electron. · · · It is not unreasonable to believe that the difficulties of modern quantum field theories have their cause in the inconsistencies of their classical parent theories.

Milner made a serious attempt to extend his theory into the realm of gravitation and general relativity. In doing so he made two significant remarks. One was to suggest that his scalar magnetic force might be the source of gravitational phenomena, and the other that, whereas Einstein's general relativity seeks explanations in terms of a strained space, his own theories use a stressed space. This latter idea is not new, although Einstein's ideas on the subject appear only recently to have been frequently quoted. Thus Max Born (1969) quotes Einstein's argument that the lemma that "the movement of mass points is determined by the geodetic lines of a space-time world," follows from the postulate that "the metric of this world satisfies Einstein's field equation." Chiu and Hoffmann (1964) put this even more clearly when they say "in the Einstein field equations the geometry is determined from the field equations, whereas in special relativity the geometry is a priori restricted to that which is Lorentz-invariant."

The concept of stressed space has been found to be rather curiously supported by J. J. Thomson's (1921) discovery that although there is no momentum, there is a net moment of momentum in the presence of static electric and magnetic poles. Of course Thomson was thinking in terms of a stressed ether, but, for me at any rate, Einstein resolved this question of what is stressed, when he wrote (Samuel, 1951)

> The programme of the field theory has the great advantage that it makes a separate concept of space (as distinguished from space-content) superfluous. The space is then merely the four-dimensionality of the field, and no longer something existing in isolation.

Thus both Einstein and Milner are working with fields: If one uses the auxiliary concept of space as an aid to reasoning, then Einstein's is strained and non-Euclidean and Milner's is stressed and Euclidean.

4. *The Essence of Milner's Theory and its Development*

The strength of Milner's technique to a "naive realist" is that he works with quaternions which deal with physical concepts that have been prevalent since Faraday and Maxwell. It is, however, somewhat confusing to the reader of his posthumous works (1960, 1963) to find that he starts with a dissertation on Eddington's E numbers which, to most people, savour of mathematical mysticism. Milner attempted to discover a set of simple rules governing the behavior of E numbers and was led in the process to split a 4 x 4 matrix into the sum of 4 x 4 matrices. This sum, when treated as a column vector, was found to obey the rules of quaternion algebra. He then showed that by combining this new assembly with its transpose and any 4 x 4 square matrix he had devised a new matrix which he called the e number or e transpose of the square matrix. Further the sum of the squares of the terms of the e number and the square matrix, which he called their resultant magnitude, were the same.

Then, in Milner's own words (1960),

> · · · the basic assumption of the analysis · · · is that the rest-mass density scalar of fundamental matter can be decomposed into, or alternatively regarded as the resultant of, the component terms of two matrices (zz^*) and Z, the first denoting in 4-space a complex vector field, and the second its energy–momentum-flux mechanical properties. The assumption is confirmed by evaluating Z in detail, when a correspondence between it and the similar matrix of standard electromagnetic theory becomes apparent.

Thus, as with Maxwell, the mathematics may be said to have preceded the theory. Z is of course the e number of (zz^*), and, having argued for the conservation of Z "by applying the principle of stationary action" (loc. cit.) he obtains his extended electromagnetic equations.

It is at this point that, following a criticism of Kilminster's (1963), I introduced a modification to Milner's theory which will be presented below. Milner left his theory at the point where he had shown that various configurations of the fields were stable and might be used to describe some of the fundamental particles. He also suggested that his scalar magnetic force might be the basis of gravitational phenomena, but he did little more than attempt to reinterpret relativity theory in terms of his own ideas.

The quantization of Milner's theory has made possible the construction of models of the fundamental particles from which some of their characteristics such as mass and magnetic moment may be accurately calculated. The basic particles are the electron and proton, and from these all others are constructed. The models of the electron and proton give the correct magnetic moment, but an important discovery seems to be that the proton's magnetic field is not wholly due to its spin but is due to magnetic charge. This theory is made more plausible by the kaon models, which are based on cancellation of such a charge, and also by the new ψ particle, which is similarly based on such cancellation in the model of the Λ particle.

It so happens that the scalar electric field which is responsible for magnetic "charge" may also prove to be an antigravitational force. Thus the ψ-particle experiment may prove to be the first step towards the detection of the magnetic monopole and antigravitational forces.

Finally the development of gravitational theory arises from consideration of the behavior of Milner's extended field equations in vacuo as distinct from inside a particle. It is found that by incorporating the measured value of Newton's gravitational constant, the radius of the observable universe may be calculated within 5% of the measured value and a decaying constant of gravitation of 1.4 parts in 10^{10}/yr., which compares with an estimate of 1 part in 10^{10}/yr. made from the measured annual recession of the moon.

5. *Summary of Milner's Results*

Milner's theory may be very quickly summarized (Kilmister, 1969, p.c) (though such a summary omits the important question of motivation, for which the reader must turn to Milner's works).

Maxwell's equations can be written, in convenient form, as

$$F^{ij}{}_{,j} = J^i \qquad (5.1)$$

$$\tfrac{1}{2}\epsilon_{ijkl}F^{ijk} = 0$$

where commas denote differentiation, $i, j, k, \cdots$ take values 1, 2, 3, 4, the coordinates are $x^1 = x$, $x^2 = y$, $x^3 = z$, and $x^4 = t$, and

$$F^{ij,k} = \eta^{kl}F^{ij}{}_{,l}$$

Milner proposes to modify those by writing

$$F^{ij}{}_{,j} = J^i = R^i + \phi^{,i} \qquad (5.2)$$

$$\tfrac{1}{2}\epsilon_{ijkl}F^{ij,k} = K_l = S_l + \psi_{,l}$$

Here ϕ, ψ are new scalar fields and K_l (with the obvious interpretation of magnetic charge and current) is inserted, since at this stage it is not obvious whether (e.g.) the measured electric currents are to be J^i or R^i.

In addition to these equations, Milner adds four other "which have to be satisfied in order that the mechanical properties of matter shall be properly expressed by the field." These have the form

$$\tfrac{1}{2}\epsilon_{ijkl}S^iF^{jk} + F_{li}R^i = \phi R_l + \psi_l \qquad (5.3)$$

This completes the description of the general form of Milner's theory. It is evidently a highly nonlinear modification of electrodynamics, and in order to derive definite results from it Milner introduces some additional special assumptions in particular cases. It is these additional assumptions that we wish to modify.

With an extension of the usual correspondence

$$(F^{41}, F^{42}, F^{43}) = \mathbf{e}, \qquad J_4 = j_t, R^4 = r_t, \qquad \phi = -e_t$$

$$(F^{23}, F^{31}, F^{12}) = \mathbf{h}, \qquad (J^1, J^2, J^3) = \mathbf{j}, \qquad (R_1, R_2, R_3) = \mathbf{r}, K_4 = -k_t$$

$$S_4 = -S_t,\ \psi = h_t, \qquad (K_1, K_2, K_3) = \mathbf{k}, \qquad (S^1, S^2, S^3) = \mathbf{s}$$

$$(5.4)$$

(where care has been taken to agree with Milner's notation) these equations take the form in Milner's quaternionic notation:

$$(\partial_{ct}e_t + \partial_x e_x + \partial_y e_y + \partial_z e_z - \gamma_t) + i(\partial_{ct}h_t + \partial_x h_x + \partial_y h_y + \partial_z h_z - s_t) = 0$$

$$i(\partial_{ct}e_x + \partial_x e_t - \partial_y h_z + \partial_z h_y - \gamma_x) - (\partial_{ct}h_x + \partial_x h_t + \partial_y e_z - \partial_z e_y - s_x) = 0$$

$$i(\partial_{ct}e_y + \partial_y e_t - \partial_z h_x + \partial_x h_z - \gamma_y) - (\partial_{ct}h_y + \partial_y h_t + \partial_z e_x - \partial_x e_z - s_y) = 0$$

$$i(\partial_{ct}e_z + \partial_z e_t - \partial_x h_y + \partial_y h_x - \gamma_z) - (\partial_{ct}h_z + \partial_z h_t + \partial_x e_y - \partial_y e_x - s_z) = 0 \tag{5.5a}$$

or by taking resolutes and writing in the usual three vector form

$$\begin{aligned} \operatorname{div} \mathbf{e} &= j_t = r_t - \frac{1}{c}\frac{\partial e_t}{\partial t} \\ i\left(\frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} - \operatorname{curl} \mathbf{h}\right) &= ij = i(\mathbf{r} - \operatorname{grad} e_t) \\ i \operatorname{div} \mathbf{h} &= ik_t = i\left(s_t - \frac{1}{c}\frac{\partial h_t}{\partial t}\right) \\ -\left(\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} + \operatorname{curl} \mathbf{e}\right) &= -\mathbf{k} = -(\mathbf{s} - \operatorname{grad} h_t) \end{aligned} \tag{5.5b}$$

Together with

$$h_t s_t + \mathbf{e}\cdot\mathbf{r} + e_t r_t + \mathbf{h}\cdot\mathbf{s} = 0 \tag{5.6a}$$

$$h_t\mathbf{r} + e_t\mathbf{s} + r_t\mathbf{h} + s_t\mathbf{e} = \mathbf{e}\times\mathbf{s} - \mathbf{h}\times\mathbf{r} \tag{5.6b}$$

Milner's additional assumptions are contained in his derivation of the wave equation:

$$\Box^2(e + ih) + x^2(e + ih) = 0 \tag{5.7}$$

where x is a constant and $\Box^2 = \nabla^2 - (1/c^2)/(\partial^2/\partial t^2)$.

In order to examine this and the consequent relationship between the source terms and the field variables, the equations will be recast in quaternion form.

6. *Relationship between Sources and Field Variables*

Following an argument by Kilmister (1973 pc), (5.5) may be written in quaternion notation as

$$\bar{D}Z = W \tag{6.1}$$

where

$$D = \frac{1}{c}\frac{\partial}{\partial t} - i\nabla$$

$\bar{D}$ is its conjugate, and

$$Z = e + ih$$

$$e = e_t + i(\mathrm{i}e_x + \mathrm{j}e_y + \mathrm{k}e_z)$$

$$h = h_t + i(\mathrm{i}h_x + \mathrm{j}h_y + \mathrm{k}h_z)$$

Suppose W to be a general linear function of the field vectors, so that

$$W = \Sigma_i U_i Z t_i \qquad (6.2)$$

The transformed equations will be

$$W' = \Sigma_i U_i' Z' t_i'$$

Now under a Lorentz transformation

$$q \to q' = a\, q\, a^+ \qquad (/a/ = 1)$$

we have

$$D \to D' = a\, D\, a^+$$

$$\bar{D} \to \bar{D}' = a^* \bar{D}\, \bar{a}$$

where $^+$ denotes the Hermitean conjugate. So if, correspondingly, we assume

$$Z' = a\, Z\, b \qquad (6.3)$$

it follows that

$$\bar{D}Z \to (\bar{D}Z)' = a^* \bar{D} \bar{a} a Z b$$

so that

$$W \to W' = a^* W b$$

Whence

$$a^* UZtb = U'aZbt'$$

Consequently

$$U' = a^* U\bar{a} \qquad \text{and} \qquad t' = \bar{b}tb \qquad (6.4)$$

Kilminster comments that the U is clearly a four-vector and the t a six-vector. We have to take account of the quadratic condition established by Milner, Eqs. (5.3) and (5.6). These take the simple form, in quaternion notation,

$$Z^+W + W^+Z = 0 \qquad (6.5)$$

i.e.,

$$Z^+UZt + (UZt)^+Z = 0$$

or

$$Z^+UZ + t^+Z^+U^+Zt^{-1} = 0 \tag{6.6}$$

It is at this point that a choice must be made for U and t. Milner effectively made them scalar quantities so that

$$Ut = x_m$$

a constant with dimensions of an inverse length. Physically a six-vector suggests rotation, but if it is to commute with Z^+U^+Z its vector parts must be somehow restricted. One obvious solution to this problem is to ensure that the vectorial parts of t are orthogonal to those of Z^+U^+Z so that vector multiplication is zero. But to do this the fields have somehow to be anchored, at least relatively to one another, in space, and this implies sources. Clearly then such a solution may well be applicable to problems involving the interaction of particles. Note that U and t are independent of e and h for a linear theory so that the orthoganality of **e** and **h** is of no help.

For the present we will examine the case of the field inside the particle and take a simple case of reducing all components of t to zero, except the scalar part. It will then commute with Z^+U^+Z and a possible solution of (6.6) is

$$U = U^+ \qquad \text{and} \qquad t^+ = -t = -iA \tag{6.7}$$

where A is a real number.

The condition $U = U^+$ requires that a four-vector should be physically real, B say. So we may write

$$W = UZt = iABZ = ixz \tag{6.8}$$

where $x = (x_t + i\mathbf{x})$.

Substituting (6.8) in (5.5), where

$$r + is = UZt = ix(e + ih)$$

$$e = (e_t + i\mathbf{e}) \qquad \text{and} \qquad h = (h_t + i\mathbf{h})$$

we have

$$ixe = ix_te_t - i\mathbf{x}\cdot\mathbf{e} + \mathbf{x}e_t - x_t\mathbf{e} + i\mathbf{x}\times\mathbf{e} \tag{6.9a}$$

and

$$-xh = -x_th_t - \mathbf{x}\cdot\mathbf{h} - i\mathbf{x}h_t - ix_t\mathbf{h} + \mathbf{x}\times\mathbf{h} \tag{6.9b}$$

so

$$\frac{1}{c}\frac{\partial e_t}{\partial t} + \operatorname{div}\mathbf{e} = x_th_t + \mathbf{x}\cdot\mathbf{h} \tag{6.10a}$$

$$\frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} + \operatorname{grad} e_t - \operatorname{curl}\mathbf{h} = -\mathbf{x}h_t - x_t\mathbf{h} - \mathbf{x}\times\mathbf{e} \tag{6.10b}$$

$$\frac{1}{c}\frac{\partial h_t}{\partial t} + \text{div}\,\mathbf{h} = x_t e_t + \mathbf{x}\cdot\mathbf{e} \tag{6.10c}$$

$$\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} + \text{grad}\,h_t + \text{curl}\,\mathbf{e} = -\mathbf{x}e_t - x_t\mathbf{e} + \mathbf{x}\times\mathbf{h} \tag{6.10d}$$

These equations must clearly reduce to Maxwell's when the vectorial field sources and the time variations in e_t and h_t are reduced to zero. Then

$$j_t = x_t h_t \quad \text{and} \quad \mathbf{j} = \mathbf{x}h_t \tag{6.11a}$$

$$k_t = x_t e_t \quad \text{and} \quad \mathbf{k} = \mathbf{x}e_t \tag{6.11b}$$

and it must follow from the relationship between j_t and $\mathbf{j}$ and k_t and $\mathbf{k}$ that

$$\mathbf{x} = (\mathbf{v}/c)x_t \tag{6.12}$$

where $\mathbf{v}$ is a velocity vector.

This leads to

$$x_t\mathbf{h} = -\mathbf{x}\times\mathbf{e} \quad \text{and} \quad x_t\mathbf{e} = \mathbf{x}\times\mathbf{h} \tag{6.13}$$

whence the well-known result

$$\mathbf{h} = -(\mathbf{v}/c)\times\mathbf{e} \quad \text{and} \quad \mathbf{e} = (\mathbf{v}/c)\times\mathbf{h} \tag{6.14}$$

which ensures the constancy of the velocity of light, since motion only results in equal rotation of the e and h fields.

Substituting (6.13) into (6.10) we have

$$\text{div}\,\mathbf{e} = j_t = x_t h_t + \mathbf{x}\cdot\mathbf{h} - \frac{1}{c}\frac{\partial e_t}{\partial t} \tag{6.15a}$$

$$i\left(\frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} - \text{curl}\,\mathbf{h}\right) = -i\mathbf{j} = i(-\mathbf{x}h_t - \text{grad}\,e_t) \tag{6.15b}$$

$$i\,\text{div}\,\mathbf{h} = ik_t = i\left(x_t e_t + \mathbf{x}\cdot\mathbf{e} - \frac{1}{c}\frac{\partial h_t}{\partial t}\right) \tag{6.15c}$$

$$-\left(\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} + \text{curl}\,\mathbf{e}\right) = -\mathbf{k} = -(-\mathbf{x}e_t - \text{grad}\,h_t) \tag{6.15d}$$

However, if we put $\mathbf{x} = 0$ in (6.10) we obtain

$$\text{div}\,\mathbf{e} = x_t h_t - \frac{1}{c}\frac{\partial e_t}{\partial t} \tag{6.16a}$$

$$\frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} - \text{curl}\,\mathbf{h} = -x_t\mathbf{h} - \text{grad}\,e_t \tag{6.16b}$$

$$\text{div}\,\mathbf{h} = x_t e_t - \frac{1}{c}\frac{\partial h_t}{\partial t} \tag{6.16c}$$

$$\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} + \text{curl } \mathbf{e} = -x_t\mathbf{e} - \text{grad } h_t \tag{6.16d}$$

Taking div of (6.16b) we obtain

$$\frac{1}{c}\frac{\partial}{\partial t}(\text{div } \mathbf{e}) + \nabla^2 e_t + x_t \text{ div } \mathbf{h} = 0$$

Whence, using (6.16a) and (6.16c),

$$-\frac{1}{c^2}\frac{\partial^2 e_t}{\partial t^2} + \frac{x_t}{c}\frac{\partial h_t}{\partial t} + \nabla^2 e_t - \frac{x_t}{c}\frac{\partial h_t}{\partial t} + x_t^{\,2} e_t = 0$$

or

$$\Box^2 e_t + x_t^{\,2} e_t^{\,2} = 0 \tag{6.17a}$$

Similarly

$$\Box^2 h_t + x_t^{\,2} h_t = 0 \tag{6.17b}$$

and

$$\Box^2 j_t + x_t^{\,2} j_t = 0 \tag{6.17c}$$

where

$$\Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

Similarly, taking curl of (6.16b) and using (6.16d) and (6.16c), we obtain

$$\boxtimes^2 \mathbf{h} + x_t^{\,2}\mathbf{h} = 0$$

and

$$\boxtimes^2 \mathbf{e} + x_t^{\,2}\mathbf{e} = 0$$

where

$$\boxtimes^2 = \text{✡} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$

and

$$\text{✡} = \text{grad div} - \text{curl curl}$$

To summarize the foregoing: we have taken Eq. (6.1) as being the most general form of the electromagnetic equations. In Maxwell's form Z is the bivector in terms of the electric and magnetic vectors. In Milner's form, on the other hand, Z is a biquaternion in terms of electric and magnetic quaternions. This introduces two new concepts, a scalar electric and a scalar magnetic field, and is the most general form of Eq. (6.1). In Eq. (6.2), however, the sources were assumed to be general linear functions of the field vectors and subjected to the necessary quadratic conditions established by Milner. This resulted in Eq. (6.6). By restricting the six-vector to its scalar parts, Eqs. (6.10) are easily derived, and by introducing the relationship between charge and current, it was found that the well-known velocity relationship between orthoganal electric and magnetic fields became a consequence of these extended electromagnetic (EM) equations. Further, certain terms drop out of (6.10) yielding (6.15). However, if one returns to (6.10) and puts the velocity equal to zero, (6.16) results and leads to the wave equations (6.17) and (6.18).

Although j_t and $\mathbf{j}$ have been used to facilitate comparison of the developed with the Maxwellian equations, it should be remembered that, so far in the analysis, densities are fluid field densities and must not be regarded in terms of a number of particles per unit volume. This aspect, which involves statistical distribution, will be dealt with later. The consequence of this restriction is, broadly speaking, that the developed, or extended, equations are so far only applicable inside a particle. In order to examine their behavior in vacuo it is necessary to know more about the nature of x and how it behaves.

To do this the equations have to be quantized and particles have to be introduced into the theory.

7. *Spherical Particle with* $\mathbf{x} = 0$ *and* $\mathrm{x}_t = const$ *and* $\mathbf{h} = 0$

To introduce the subject of particle models we will begin with the simplest case. Clearly, for constant x_t, h_t must be a solution of the wave equation (6.17b) For a spherical distribution of charge at rest

$$h_t = A(\sin x_t r)/r \tag{7.1}$$

where A is constant. The total charge in a sphere of radius r is then

$$\begin{aligned} Q &= \int_0^r 4\pi r^2 j_t \, dr \\ &= \int_0^r 4\pi x_t A r \sin x_t r \, dr \qquad \text{from (6.15a) and (7.1)} \\ &= 4\pi A x_t j_1(x_t r) \end{aligned} \tag{7.2}$$

where $j_n(\ )$ is a spherical Bessel function of the first kind of order n. At the radii at which h_t is zero we have

$$Q = (-1)4\pi^2 nA/x_t \tag{7.3}$$

The energy density at r is $\frac{1}{2}(h_t^2 + e_r^2)$, and Milner shows that the total rest energy of the system, with h_t zero at the boundary, is

$$W = 4\pi^2 nA^2/x_t = AQ_0 \tag{7.4}$$

where Q_0 is the magnitude of the charge, whether this is positive or negative. Substituting (7.2) into (7.4) gives

$$W = (Q_0^2/4\pi^2 n)x_t \qquad (7.5)$$

Thus for a particle bounded at the first zero ($n = 1$), $x_t \propto W$, and by writing $W = mc^2$ it will be seen that x_t transforms as m in a Lorentz transformation. Here we have the first clue to the behavior of x_t as a variable in space and time.

8. *The Uniformly Volume-Charged Sphere*

Consider a sphere of electric charge at rest, in equilibrium, and of uniform density. Then

$$x_t h_t = j_t = \text{const} \qquad (8.1\text{a})$$

and

$$e_r = (Q_r/4\pi r^2) = (r/3)j_t \qquad (8.1\text{b})$$

Also for zero magnetic charge

$$-x_t \mathbf{e} - \text{grad } h_t = k_t = 0 \qquad (8.1\text{c})$$

From (8.1a) we have

$$x_t = j_t/h_t$$

so that

$$\text{grad } h_t = -x_t e_r = -(j_t/h_t) r j_t/3$$

or

$$\frac{dh_t}{dr} + \frac{r}{h_t}\frac{j_t^2}{3} = 0 \qquad (8.2)$$

where x_t has been eliminated. A solution of (8.2) is given by

$$h_t^2 + \frac{r^2 j_t^2}{3} + b_1 = 0 \qquad (8.3)$$

where b_1 is a constant. At the boundary of the sphere $r = a$ and $h_t = 0$, so

$$b_1 = -\tfrac{1}{3}a^2 j_t^2$$

and finally

$$h_t^2 = \tfrac{1}{3}(a^2 - r^2) j_t^2 \qquad (8.4\text{a})$$

or

$$3h_t^2/(a^2 - r^2) = j_t^2 = \text{const} \qquad (8.4\text{b})$$

Suppose the h_t field has spin; h_t transforms as

$$h_t = \frac{h_t'}{(1 - v^2/c^2)^{1/2}} \tag{8.5}$$

so (8.4) is satisfied if

$$1 - r^2/a^2 = 1 - v^2/c^2 \tag{8.6a}$$

i.e., if

$$\frac{r^2 \omega^2 \sin^2 \theta}{a^2 \omega^2 \sin^2 \theta} = \frac{v_\phi{}^2}{c^2} \tag{8.6b}$$

or

$$v_\phi = r\omega \sin \theta \qquad \text{and} \qquad \omega \sin \theta = c/a = \text{const} \tag{8.6c}$$

Thus the "spin" is composed of spherical shells with constant v_ϕ proportional to the radius r from the center.

This model will later be used to describe the electron, and it will be shown that it has the correct magnetic moment and spin.

9. *Quantization of Action in the Milner H Atom*

So far the theory has been entirely classical in the sense that there is no quantization of action. Quantization of action will now be demonstrated for a classical Milner-type model of the H atom.

A detailed construction of models of some of the fundamental particles will be reserved for a later section. However, before constructing a model of the H atom, some reason should perhaps be given why the extended electron and proton do not annihilate one another in the Milner system. The clue to the reason for this is to be found in the manner in which their magnetic moments differ, for while that of the electron is reasonably close to that for a spinning sphere of uniform density electric charge, that of the proton is very far removed from such a value. This may be explained by the absence of magnetic charge in the electron and its presence, in dipole form, in the proton. Thus the fields composing the two particles are entirely different in nature as well as magnitude and they cannot annihilate each other as they do in the case of the positron and electron. The detailed structure of the proton and electron will be examined later with the help of the methods about to be detailed below. Meanwhile it is only necessary to note that, in a model of the H atom, the proton may be regarded as a point compared with the electron.

We postulate a model of the H atom, in the ground state, in whcih the proton is a point charge situated at the center of a spinning, negatively, uniformly volume-charged spherical cloud, the spin being of the form discussed previously. Suppose this atom to be subjected to incoming radiation of angular frequency ω. (The effect on the magnetic fields of the atom may be neglected, in this case, since, being dipolar, they will tend to rotate about the center in synchronism with the radiation frequency and there will be no relative motion, due to this cause, between the proton and electron.)

The effect of the electric field will be twofold: It will polarize the atom, i.e., it will separate the centers of mass of the proton and electron in the direction of the field, besides, of course, disturbing the electron cloud. It will also cause the center of mass (c.m.) of the electron to rotate about the proton with an angular frequency ω. However, the electron will also possess a natural frequency, ω_p at which it would oscillate, if the amplitude were small, with the proton as fulcrum, if ω were zero.

This natural frequency of the electron may be likened to a collision frequency between the proton and electron cloud, for it results in a transfer of energy and momentum into directions perpendicular to that of the incoming radiation. Thus the internal energy of the electron cloud will be increased and it will adopt a configuration consistent with (6.17c):

$$\Box^2 j_t + x_t^{\,2} j_t = 0$$

The solutions of this equation are the same as those that yield the radiation pattern of a linear oscillator or radiating antenna (Stratton, 1941, pp. 438 ff.), which in turn are the same as the probability density factor for the H atom (White, 1931). This, of course gives the classical explanation of the photon, for whereas in an antenna the radiation lobes are produced by a linear current in the rod conductor, by the reciprocity theorem, the current occupying the same shaped lobes in the H atom must give rise to radiation occupying what would have been the conductor of the antenna. Thus the radiation cannot diffuse but must occupy a straight cylinder cut into lengths (i.e., quanta) by the starting and stopping of the radiation as the cloud moves from one eigenstate to the next.

To obtain a quantitative value for these quanta, we return to the behavior of the perturbed cloud under radiation. The path of the electron's c.m. may be thought of as a precessing ellipse, similar to Sommerfeld's well-known rosette orbit (Born, 1951; Sommerfeld, 1934). The equation for an ellipse may be written (Cayley, 1961)

$$(ds/d\theta)^2 = \tfrac{1}{4}a^2(1 - k^2 \sin^2 \tfrac{1}{2}\theta) \tag{9.1}$$

where $k^2 = (a^2 - b^2)/a^2$ and the coordinate axes pass through the center of the ellipse. Here a is the semimajor axis, b is the semiminor axis, and $\sqrt{a^2 - b^2}$ is the distance of the focus from the center and is thus the polarization distance.

It follows from the geometry of the rosette that the k^2 is the ratio of the rotational energy contained in the elliptic motion to the total rotational energy. The first can only be the energy imparted to the electron by moving its c.m. through twice (since the field reverses) the distance of polarization, while the second is that imparted to the electron in moving around a circle of radius equal to the distance of polarization. Thus

$$k^2 = 1/\pi$$

10. *Natural Period of the Electron*

Suppose a rotating pendulum, of mass m_e and length l, to be rotating about the proton as fulcrum, in a constant uniform field of force eQ and in phase with the electron's c.m. having the same maximum kinetic energy. Its equation of motion may be written (Milne-Thomson, 1950)

$$(d\phi/dt)^2 = \omega_p{}^2(1 - k_2{}^2 \sin^2 \tfrac{1}{2}\phi) \tag{10.1}$$

where $k_2{}^2 = 4(\mathrm{e}Q/m_e l\omega_p{}^2)$ and ϕ is measured about the focus, so that

$$r \cos\theta - l \cos\phi = ka$$

The maximum kinetic energy must have been derived from the applied field, hence

$$\tfrac{1}{2}m_e l^2 \omega_p{}^2 = 2\pi l\mathrm{e}Q \tag{10.2}$$

so that

$$k_2{}^2 = 1/\pi = k^2$$

It may be shown that, if $l = a$, then $r\, d\theta/dt = b\, d\phi/dt$, so that the period of the electron around the elliptical orbit may be calculated from that of the pendulum.

Then (Milne-Thompson, 1950)

$$T = 4K/\omega_p$$

where K is, as before, the "complete elliptic integral of the first kind" with modulus $1/\pi$.

11. *Change of Shape of the Electron Cloud and Absorption of Energy*

Insofar as the proton may be regarded as having infinite mass, pure rotation of the electron about it will result in the energy being stored entirely in the rotation, provided the cloud can adopt a configuration compatible with absorption of the incoming radiation; that is, the system may be likened to the complement of a receiving antenna absorbing radiation, where the cloud adopts a configuration that fills the radiation pattern of the antenna and the incoming radiation occupies the space of the antenna itself.

The mechanism by which the cloud changes shape is its interaction with the proton, the oscillation at its natural frequency resulting in a transfer of momentum to the directions perpendicular to the direction of the applied field. Energy will continue to be absorbed in this way until the cloud reaches its next stable shape as determined by the wave equation.

The transfer of momentum per oscillation is given by

$$p = 2m_e a(\omega_p/4K) \tag{11.1a}$$

The rate of transfer is

$$\dot{p} = 2m_e a(\omega_p/4K)\nu \tag{11.1b}$$

where

$$\nu = \omega/2\pi$$

Hence the energy transfer per oscillation of the field (i.e., two revolutions of the cloud) is

$$2\dot{p}2\pi a = a^2 m_e(\omega_p/K)\omega \tag{11.1c}$$

12. *Planck's Constant*

The difference in energy between the two stable states is obtained by observing that this will be the result of the interaction of the proton *once* with the whole of the electron mass. During a complete rotation about the proton, the major axis of the ellipse covers an area πa^2. It follows that the limiting condition of only one interaction requires (11.1c) to be multiplied by $L^2/\pi a^2$, where L is the unit of length in the chosen units of working.

The velocity of the c.m. at its apogee is $a\omega_p$, so that the same limit requires (11.1c) also to be divided by the number of rotations of the major axis per unit time, i.e., $\nu_p T$, where T is the unit of time.

Thus we obtain for the equation of energy absorbed

$$W_0 = 4\,\frac{\dot{p}}{\nu_p}\frac{L^2}{T} = \frac{4\pi m_e}{K}\,\omega\,\frac{L^2}{T} \tag{11.1d}$$

This leaves the absorbed energy proportional to the driving frequency. Equating the constant of proportionality to Planck's constant yields, after correction for the finite mass of the proton,

$$2\pi h = \frac{4\pi}{K} m_e\left(1 + \frac{m_e}{M_p}\right)^{-3} \cdot \frac{L^2}{T} \tag{12.1}$$

Taking

$$\frac{m_e}{M_p} = \frac{1}{1836.093}, \qquad K = 1.7247936$$

$$\frac{2\pi\hbar}{m_e} = 7.27382644 \text{ cm}^2 \text{ sec}^{-1} \tag{12.2}$$

which agrees exactly with the value given by Dicke (1957). The deviation from later values, e.g., 7.27359 (Dicke, 1963) is probably due to measurements taken at higher voltages, which, as shown below in the discussion of the experimental values for the fine structure constant, would, on the present theory, be expected to give lower values for $\hbar$.

13. *The Fine Structure Constant*

Sommerfeld (1934) showed that the introduction of the Lorentz transformation of mass into the equations of motion of the electron in the Bohr atom results in a Thomas precession (Cerbin & Stehle, 1950) as follows:

$$\gamma\omega\tau = 2\pi \tag{13.1a}$$

where

$$\gamma = 1 - Z^2Q^4/p^2c^2$$
$$= 1 - (\hbar^2/p^2)Z^2\alpha^2 \tag{13.1b}$$

in which p is the angular momentum, Z the charge number, and α the fine structure constant (f.s.c.),

$$\alpha = Q^2/\hbar c$$

Rearranging (13.1b) we obtain

$$p^2\gamma^2 = p^2 - \alpha^2\hbar^2Z^2$$
$$= (n_\phi{}^2 - \alpha^2Z^2)\hbar^2 \tag{13.1c}$$

where n_ϕ is the azimuthal quantum number. Comparing with the model of the H atom, clearly

$$\gamma = 2\pi/4K$$

The total rotational energy is $\frac{1}{2}p^2/m_e$. Following Sommerfeld's formula, we divide this into two parts. The precessional energy,

$$W_p = \tfrac{1}{2}(p^2/m_e)\gamma^2 \tag{13.2}$$

and the spin energy, which is the rotational energy of a spinning sphere having a diameter equal to the root mean square value of the total excursion of the center of mass of the electron due to the radiation field. That is, its radius is $a/\sqrt{2}$, where $2a$ is the major axis of the ellipse. In center of mass coordinates, which takes the rotational energy of the proton in the radiation field into account, spin energy is given by

$$W_s = \frac{1}{2}\frac{p^2}{m_e}\gamma^2\left[\left(\frac{I}{m_ea^2}\right)^2\left(1 + \frac{m_e}{M_p}\right)^2\right]^3 \tag{13.3}$$

where I is the moment of inertia of the sphere described above. Comparing with Sommerfeld's formula (13.1) we have

$$\alpha = \frac{2\pi}{4K}\left(1 + \frac{m_e}{M_p}\right)^3\left(\frac{I}{m_ea^2}\right)^3 \tag{13.4}$$

Inserting the values of K amd m_e/M_p as before and $I/m_ea^2 = 1/5$ we have

$$\alpha = 7.29764 \times 10^{-3} \qquad \text{or} \qquad \alpha^{-1} = 137.0306 \tag{13.5}$$

14. *Comparison with Measurement*

The numerical value of the fine structure constant obtained above is larger than the currently accepted value. However, as the voltage at which measurements are made is increased, the radiation wavelength will correspondingly decrease, according to both accepted theory and the present model, where the energy absorbed, and therefore radiated, is proportional to frequency, at least until the wavelength becomes of the order of the diameter of the electron cloud. When, for example, these are equal--as for an incident energy of 0.5 MeV--the effective electric field is reduced in value to the average of its root mean square (rms) value. This reduces the modulus of K in the same ratio, i.e., from $K = 1.7247936$ to $K = 1.7012489$ or 1.636%. An estimate made from Cohen and du Mond's (1957) data makes the change in $\hbar$ about 1.5%.

A determination of the fine structure constant therefore, which depends upon the direct or indirect measurement of $\hbar$, should give a voltage dependent result that is roughly proportional to the energy of radiation. This is supported by Cohen and du Mond (1957), who present a graph of mass x-ray spectrometer determinations of $2\pi\hbar c^2/Q$, in which the discrepancies are plotted against the voltage at which the measurements are made. The graph consists of a straight line that intercepts the discrepancy voltage axis at about 0.3 V. From the lowest voltage measurements, given by them, this apparently corresponds to a deviation of about 40 ppm from the least squares adjusted value for α^{-1} of 137.03777, the value used by them (1953) in these calculations. The value calculated above diverges by 52 ppm from this, i.e., agrees with experiment on this interpretation to about 12 ppm.

Further support for the "zero voltage" value is given by the measurement of $Q/\pi\hbar$ using the ac Josephson effect, by Parker et al. (1967), who would give α^{-1} the value 137.0359(3). This assessment involves a number of other constants, and a comparison of their value of $Q/\pi\hbar$ with the calculated value of $\hbar/m$ using their value of $m = 9.10965(14) \times 10^{-28}$ g gives an agreement within 12 ppm as before, in which the calculated is the lower value. (Although this agreement appears to be dependent upon the value selected for m_e, it is in fact consistent with the other physical constants, since they raised both Q and m by 63 ppm above Cohen and du Mond's values.) A later measurement of $Q/\pi\hbar$ by Harvey et al. (1970) brings the agreement to within 7 ppm.

15. *Particle-Wave Equations*

With the aid of an H atom model we have established that radiation energy is proportional to frequency and, by assuming that the constant of proportionality is Planck's constant, have obtained a numerical value of $\hbar/m_e$, where m_e is the mass of the electron. Further, we have obtained a calculated value for the fine structure constant following Sommerfeld's model, in which α is the ratio of spin energy to rotational energy at the particle's radius. Historically quantum theory began at a similar point, namely, the observation that radiation from the H atom is quantized and statistical; but this involves the assumption that all radiation is quantized. Our demonstration so far only applies to the radiation field from a particle constrained by central forces. This is strictly analogous to the position at the beginning of quantum theory except that, then, the evidence being experimental, there was little to suggest that the concept of continuous fields was only an approximation--an idea which, surprisingly enough, seems only to have been seriously considered half a century or so later (Kilmister, 1971). The manner of calculation of $\hbar/m_e$ in the present theory, however, strongly suggests that the fields are continuous, and it becomes necessary to see whether quantization arises naturally or has to be imposed by some assumption or assumptions.

It will now be shown that quantization enters the theory through a physical interpretation of Milner's constant, and that Planck's constant is part of the mathematical description of the electron. This will be done by constructing a classical analog of the de Broglie wave equation.

The algebraic relation between Schrödinger and de Broglie waves on the onehand and Milner's waves on the other is shown by the following analysis.

A traveling wavesolution of Milner's equation for the scalars e_t and h_t

$$\Box^2\Gamma + x^2\Gamma = 0 \tag{15.1a}$$

is clearly

$$\Gamma = \Gamma_0 \exp i(x_1 y - \omega_1 t) \tag{15.1b}$$

and substituting this into (15.1a) we get

$$-x_1^2 + \omega_1^2/c^2 + x^2 = 0 \tag{15.2}$$

writing $\omega_1/x_1 = v$, the velocity of the wave, we have

$$x_1 = x(1 - v^2/c^2)^{-1/2} \tag{15.3}$$

and

$$\frac{\partial^2\Gamma}{\partial y^2} + \frac{1}{v^2}\frac{\partial^2\Gamma}{\partial t^2} = 0 \tag{15.4a}$$

and

$$\frac{\partial^2\Gamma}{\partial y^2} + x_1^2\Gamma = 0 \tag{15.4b}$$

Thus Milner's wave equation represents a wave traveling with a velocity less than that of light. The waves of quantum theory (Schrödinger, de Broglie, and Klein-Gordon), however, have velocities greater than that of light, and Milner shows that one cannot convert his equation to this form simply by making x imaginary. The physical reasons for this may be seen by writing the wave equation in the form

$$\frac{\partial^2\Gamma}{\partial y^2} - \frac{1}{U^2}\frac{\partial^2\Gamma}{\partial t^2} = 0 \tag{15.5a}$$

or

$$\frac{\partial^2\Gamma}{\partial y^2} - x_2^2\Gamma = 0 \tag{15.5b}$$

where U is now a phase velocity greater than that of light. With a similar substitution to (15.1b) we obtain the solution

$$-x_2^2 + \omega_2^2/c^2 - x^2 = 0 \tag{15.6}$$

Following Fermi (1951) we multiply (15.6) throughout by $\hbar^2$ and compare with the well-known relation

$$W = (c^2p^2 + m_0^2c^2)^{1/2} \tag{15.7}$$

This yields the following correspondences: W, the total energy, equals $\hbar\omega_2$, p, the momentum, equals $\hbar x_2$, and m_0, the rest mass, equals $\hbar x^2/c$. Making the same comparison with Milner's equation, i.e., (15.2) with (15.7) we have

$$W = \hbar x_1 c$$

$$p = \hbar\omega/c$$

and

$$m_0 = \hbar x/c$$

as before. It follows that, if the result obtained in (11.1), namely, that the energy of the radiated or absorbed wave is proportional to its frequency, is to be expressed in a wave equation, we must choose one of the forms of (15.5b) rather than (15.4b), i.e., an equation of the form

$$\Box^2\Gamma - x_e^2\Gamma = 0 \tag{15.8}$$

In order to construct such an analog of de Broglie's equation, it has to be borne in mind that Milner's equation refers to fields inside the particle, whereas de Broglie's refers to the movement of the particle as a point charge. The constructed wave equation will therefore be built from a summation of Milner equations.

Now Milner shows (1963, sec. 7.5.; 1960, pp. 202 et seq.) that the time rate of charge of h_t^2 and e_t^2 gives rise to mechanical forces. Further, we have seen from (7.5), if we assume $W = mc^2$, and in (15.1a) above that x_t is proportional to particle mass. With this in mind we shall assume that a cluster of h_t waves of equal amplitude but varying x_t (which we will write as h_n and x_n, respectively) will obey the Maxwell–Boltzmann distribution law.

For the electron, we will assume the model of a uniformly volume-charged sphere obeying Eq. (8.4a). Substituting (8.1a) we have

$$x_t = [3/(a^2 - r^2)]^{1/2} \tag{15.9}$$

Thus the electron has shells of constant x_t that varies from zero to infinity. Assume an infinite set of h_t waves, of equal amplitude h_n but wavelength $1/x_n$ varying from zero to infinity, propagating along the Y axis of the electron. At the origin we have from (8.4a)

$$\tfrac{1}{2}h_t^2 = \tfrac{1}{6}j_t^2a^2 \tag{15.10}$$

Assuming a like relationship for the h_n waves we have

$$\tfrac{1}{2}h_n^2 = \tfrac{1}{2}j_t/x_n^2 = \tfrac{1}{6}j_t^2b^2 \tag{15.11}$$

To find b introduce the Maxwell–Boltzmann relation at the origin. Then

$$h_t = \frac{j_t a}{\sqrt{3}} = \frac{j_t}{\sqrt{3}} \int_0^\infty \exp \beta\left(-\frac{j_t^2}{x_n^2}\right) d\left(\frac{1}{x_n}\right) \tag{15.12a}$$

$$= j_t\left(\frac{\pi}{3\beta}\right)^{1/2} \tag{15.12b}$$

where

$$\beta = \frac{1}{kT} = \frac{1}{h_n^2} = \frac{3}{j_t^2 b^2}$$

whence

$$a^2 = \pi b^2 \tag{15.13}$$

We have now a geometric picture of a cluster of waves propagating along the Y axis, the sum of their x_n's at each point equaling the x_t of the spherical wave. Each x_t occupies a surface area $4\pi r^2$, but, since the h_n waves are linear, each sum of x_n's represents a surface y^2. To obtain an equivalent x for the wave group we integrate over the Y axis from $y = 0$ to $y = b$ equivalent to 1/4 wavelength of the group wave. Thus writing $y = b \sin\theta$

$$\frac{4}{\lambda_e} = 4x_e = \frac{1}{a}\int_0^{\pi/2} \frac{d\theta}{[1-(b^2 \sin^2\theta/a^2)]^{1/2}} \tag{15.14a}$$

or

$$x_e = \frac{1}{4a}\int_0^{\pi/2} \frac{d\theta}{[1-(1/\pi)\sin^2\theta]^{1/2}} \tag{15.14b}$$

$$= K/4a$$

where K is the complete elliptic integral of the first kind with modulus $1/\pi$. For a spherical particle of radius a and constant x, we have from (7.1)

$$x_t = 1/2a$$

Hence

$$x_e = \tfrac{1}{2}Kx_t \tag{15.15}$$

To identify x_e with $mc/\hbar$ we note that Stratton (1941) has shown that, for a good conductor, Maxwell's equations involve two dimensionless constants:

$$b_1 = \mu\epsilon(L/T)^2 \tag{15.16a}$$

$$b_2 = \mu\sigma L^2/T \tag{15.16b}$$

where μ is the permeability of the medium and ϵ its permittivity. Thus $(\mu\epsilon)^{1/2}$ has dimensions of reciprocal velocity and $\mu\sigma$ of moment of momentum divided by mass. The two together have dimensions of reciprocal length, as has x.

Furthermore x_t in Eq. (6.8) was seen to be composed of two constants A and B, and (6.10) gives

$$\mathrm{x} = \mathrm{v}/c\, x_t$$

So we may tentatively put $A = c$ and B equal to a universal constant having dimensions L^2/T.

Now J. J. Thomson (1921) showed that in the case of an electric and a magnetic monopole, Q and G, separated by a distance r, there is a net zero momentum in space but a moment of momentum

$$GQ/c$$

even when the charges are stationary. For the electron we have an electric monopole and magnetic dipole so that we might take

$$B = 2GQ/m_e c \tag{15.17}$$

with dimensions L^2/T. But from (12.1), without the mass correction, we have

$$\frac{\hbar}{m_e} = \frac{2}{K}\frac{L^2}{T} \tag{15.18}$$

and from (15.13)

$$= \frac{x_t}{x_e}\frac{L^2}{T} \tag{15.19}$$

In the absence of magnetic poles we may put $B = 1$ so $x_t = C$, but in the presence of a magnetic dipole we would choose $B = m_e/\hbar$ so that

$$x_e = \tfrac{1}{2}Kx_t = m_e c/\hbar \tag{15.20}$$

Substituting in (15.17) gives

$$\hbar = 2GQ/c \tag{15.21a}$$

$$= Q^2/\alpha c \tag{15.21b}$$

by substitution of the value of Dirac's (1948) monopole (see below). Thus the combination of monopole and dipole would appear to be fundamental in nature.

16. *Models of Some of the Fundamental Particles*

It has been shown how stable models of particles may be constructed consisting either of a sphere of electric charge of uniform density or a sphere of charge in which x_t is constant throughout. These ideas will now be extended to construct models of some of the fundamental particles. No rule has yet been established for doing this other than the physical interpretation of formulas that produce the right result. It is to be hoped, however, that the pattern of such models will lead to a better understanding of the principles involved so that definite rules may be laid down. This is not meant to suggest that the present rules are entirely ad hoc—they are not; but the present models are admittedly the result of much trial and error and at present can only lay claim to plausibility to the extent that they give the correct answers. Even here one has to be careful; experience teaches that for every correct forecast there are many that are almost within experimental error, and the lesson of Eddington's Fundamental Theory still applies.

16.1 *Magnetic Moment of the Electron*

It has been shown that a stable sphere of electric charge at uniform density necessarily has spin. The motion consists of concentric shells having an angular velocity about the same axis, proportional to their radii. Thus the maximum flux will occur at the poles. Simple integration over the sphere gives a pole strength equal to $\frac{1}{2}Q$, the total electric charge of the electron, and the effective magnetic charge, whose distribution must obey the wave equation, will occupy wavelength $1/x_e$. The dipole moment of the electron is thus

$$\mu_e = Q_e/2x_e + \text{terms due to precession, say } (Q_e/2x_e)(1+\alpha_0) \qquad (16.1)$$

where α_0 is a modification of α (13.4) (which contains the relativistic correction $(1 + m_e/M_p)^3$ and operates upon angular and not linear frequencies). Thus we have

$$\alpha_0 = (1 + m_e/M_p)^{-3}(1/2\pi)\alpha$$

and

$$\mu_e = \frac{\hbar Q_e}{2m_e c}\left[1 + \frac{1}{4K}\left(\frac{I}{m_e l^2}\right)^3\right] = \frac{\hbar Q_e}{2m_e c}\left[1 + \frac{2}{K}\left(\frac{m_e r^2}{5m_e l^2}\right)^3\right]$$

where $1 = \sqrt{2r}$

$$\mu_e = 1.00115955903\, \hbar Q_e/2m_e c \qquad (16.2)$$

which compares with 1.001159557 ± 0.000000030 (Matts *et al*, 1969). The simplicity of this calculation is particularly gratifying. Lai et al. (1974) at the end of their paper on the same subject remark "In addition to having a calculation which is conceptually direct, and operationally straightforward, it would be nice if it were also trivial. We consider this is not the case." I think it may be claimed for the above that it is nicely trivial as well as conceptually direct!

16.2 *Spin of the Electron*

For a spinning sphere of electric charge, the dipole moment is

$$\mu = \frac{I\omega Q}{mc} = \frac{pQ}{mc} \qquad (16.3)$$

where I is its moment of inertia, p is its spin momentum, and other terms are as before.

Applying this to the electron without precession we have from (16.1)

$$\mu_e = \frac{p_e Q_e}{m_e c} = \frac{\hbar Q_e}{2m_e c} \tag{16.4}$$

from which we have the well-known result

$$p = \tfrac{1}{2}\hbar$$

16.3 *Magnetic Monopole*

Milner's equations are symmetrical for electric and magnetic charges, so that if the magnetic counterpart of the electron were to exist, one would expect the same ratio between field produced by rotation and spin. It follows from the calculated effective pole strength of the electron that the magnetic monopole is given by

$$G = Q/2\alpha \tag{16.5}$$

in agreement with the value given by Dirac (1948).

16.4 *Magnetic Moment and Structure of the Proton*

A short calculation shows that the model of the H atom employed to calculate $\hbar/m_e$ is stable in the ground state, i.e., perturbation does not give rise to radiative disintegration, provided the mass ratio is that measured experimentally. This fact may account for the uniqueness of the uniformly volume-charged sphere as a model so far found with the aid of the present theory. The uniqueness, however, clearly does not imply the exclusion of its application to the proton, and such a model has the correct angular momentum. However, it yields the wrong magnetic moment as it stands, so we introduce magnetic charge k_t from the equations. Such a model has a further merit of meeting the objection that one would expect two concentric, oppositely charged spheres to expand and contract and thus annihilate each other.

An appropriate solution of the wave equation for the magnetic field would contain the term

$$y_{3/2}(x_p z)$$

where $y_{3/2}(\ )$ is a spherical Bessel function of the second kind of order 3/2, which ensures that the field is dipolar, and x_p is that for the proton.

The first zero is at $y_{3/2,1} = 2.798386$ instead of $x_e z = 1$ as in the case of the electron. We therefore have

$$\mu_p = [y_{3/2,1}]\hbar Q/2M_p c$$

plus correcting terms for spin, etc.

The correction now primarily consists of that due to the spinning electric charge about a fixed magnetic axis. This of course is different from precession and will not contain the $1/4K$ term in α, as in the case of the electron. The precessional correction is insignificant, and we have therefore

$$\mu_p = [y_{3/2,1}](\hbar Q/2M_p c)[1 - 2(I/md^2)^3] \qquad (16.6)$$

where d is the diameter of the proton and the factor of 2 enters because $Q = 2G$, whence

$$\mu_p = 2.792789\,\hbar Q/2M_p c$$

which compares with

$$(2.792782 \pm 0.000017)\hbar Q/2M_p c$$

(*Physics Letters*, loc. cit.).

16.5 *Composite Particles*

An attempt has been made to construct models of some of the other so-called fundamental particles using only excited modes of protons and electrons. Most of these modes have a central nucleus, which is a proton, in normal or excited state, surrounded by electrons, again in various states; but there are, of course, exceptions, notably the pions and kaons.

The first particle we will consider will be the Λ particle.

16.5.1 The Λ Particle. From (7.5), the energy of a sphere of charge with constant x_e is given by

$$W = (Q^2/4\pi^2)x_e$$

Milner has shown that the corresponding energy for the uniformly volume-charged sphere of radius a is given by

$$W_0 = Q^2/5\pi a \qquad (16.7)$$

Combining these two, we obtain

$$W = (5/4\pi)x_e a W_0 \qquad (16.8)$$

Assuming both particles have proportional magnetic fields we may write

$$W_0 = m_e c^2$$

where m_e is now the relativistic mass of the electron.

Now

$$\alpha = Q^2/\hbar c$$

so

$$\alpha = (1/\hbar c)(4\pi^2 W/x_e) \qquad (16.9)$$

or

$$\hbar c x_e = (1/\alpha)(4\pi^2 W)$$

and from (16.8)

$$= (1/\alpha) 5\pi x_e a m_e c^2 \quad (16.10)$$

Now assume a fundamental wavelength

$$\lambda_t = 1/x_e = 2\pi a$$

Then

$$2\pi a x_e = 1 \quad (16.11)$$

and

$$\hbar c x_e = (5/2\alpha) m_e c^2 \quad (16.12)$$

Clearly the angular velocity of the wave

$$a\omega = a x_e c = v_\phi$$

Then

$$m_e = m(1 - v_\phi{}^2/c^2)^{-1/2} \quad (16.13)$$

where m is the rest mass of the electron. From (16.11) we have

$$m_e = m(1 - 1/4\pi^2)^{-1/2} \quad (16.14)$$

Substituting into (16.12) gives the energy of a particle of constant x_e and charge Q rotating about a center at radius a, the radius of a uniformly charged electron. Thus we have

$$\hbar c x_e = (5c^2/2\alpha) m (1 - 1/4\pi^2)^{-1/2} \quad (16.15)$$

In evaluating (16.15) we must omit the center of mass correction term from α. It may be noted that the term similarly cancels from Sommerfeld's formula (loc. cit.) If we now add the mass of a proton we obtain a mass of 1115.51 MeV, which compares with the mass of a Λ particle of 1115.6 ± 0.08 MeV (*Physics Letters*, loc. cit.) Thus the Λ particle may be likened to a Milner H atom in which the electron is rotating at a radius equal to its own radius in the ground state.

16.5.2. *The Neutron.* The mass of the neutron follows readily from the above by converting the "electron's" rotational energy to spin energy.

Thus $n = \frac{5}{2} mc^2(1 - 1/4\pi^2)^{-1/2}$ with a correction due to the precession of the "electron" cloud being turned to spin by the magnetic field of the proton and reversed.

Thus the contribution from the electron's magnetic field is reduced by $2(I/md^2)^3$ (see equation for μ_p) from $(1/4K)(I/ml^2)^3$ (see equation for μ_e).

Thus we have

$$n = \frac{5}{2} mc^2 \left(1 - \frac{1}{4\pi^2}\right)^{-1/2} \left[1 - \left(2 - \frac{2}{K}\right)\left(\frac{I}{md^2}\right)^3\right] + M_p$$

$$= 1.29356 \text{ MeV} + M_p$$

which compares with 1.2935 ± 0.0001 MeV (*Physics Letters*, loc. cit.).

16.5.3. *Other "Stable" Particles.* Models of the other "stable" particles may be set up in a similar fashion. Some of them are not fully worked out, but certain trends point to possible rules for their construction. For example the Σ^+ may be constructed from a proton and a zero-charge "electron" as follows.

Write (7.2) for Q_r in terms of trigonometric functions. Then

$$Q_r = 4\pi A x_e [-r \cos x_e r + (\sin x_e r)/x_2] \tag{16.16}$$

Compare with (7.1),

$$h_t = A(\sin x_e r)/r$$

It will be seen that at the first zero of h_t, Q_r is a maximum. Putting $Q_r = 0$, the boundary is now at

$$j_{3/2,1} = 4.493409$$

and the mass of such a particle will be

$$(4.493409/\pi) m_e$$

Substitution of this particle for the electron in the Λ particle gives a total positive charge and a mass of Σ^+ = 1189.4078 MeV [cf. 1189.40 ± 0.19 (*Physics Letters,* loc. cit.)].

The masses of Σ^0 and Σ^- may be obtained by adding 6 and 5π electron masses, respectively. Plausible arguments may be adduced to justify such formulas, but they are not sufficiently rigorous to give the necessary confidence for the construction of useful models.

16.5.4. Ξ *and* Ω^-. The situation is similar for Ξ and Ω^-, although for the latter particle it is perhaps significant that the proton mass is excited to a state similar to that of the electron in Σ^+.

16.5.5. *Kaon.* Since it is claimed for the theory that all particles are ultimately composed of protons and electrons, there is little to be gained in its presentation by constructing models except to check on its veracity. However, there remains the question of the possible detection of the h_t and e_t forces and models of the kaons, and the recently discovered ψ particles give a clue as to how this might be accomplished.

From (6.16) we have

$$\text{div}\, \mathbf{h} = k_t = x_c e_t - (1/c)(\partial e_t/\partial t)$$

Substituting $e_t = E \exp(i\omega t)$ yields

$$\begin{aligned}\operatorname{div}\mathbf{h} = k_t &= x_t e_t \pm \omega/c\, e_t \\ &= x_t e_t + \frac{x_t v/c}{(1 - v^2/c^2)^{1/2}} e_t \\ &= x_t e_t \left[1 \pm \frac{v/c}{(1 + v^2/c^2)^{1/2}}\right]\end{aligned}$$

where

$$\omega = xv = \frac{x_t v}{(1 - v^2/c^2)^{1/2}}$$

Now suppose that e_t exists but $\operatorname{div}\mathbf{h} = k_t = 0$. Then

$$v = c/\sqrt{2}$$

Thus we have a critical velocity at which magnetic charge is canceled.

Now take the mass formulas (16.12) and (16.13) and add a stationary particle of the same basic mass, i.e. (anticipating the result in our notation),

$$\begin{aligned}K &= (5/2)m[(1/\alpha) \pm 1] \\ &= (5/\sqrt{2})m_e[(1/\alpha) \pm 1]\end{aligned}$$

at the critical velocity $v = c/\sqrt{2}$; then $K = 495.6 \pm 1.8$, $K^{\pm} = 493.8$, $K^0 = 497.4$, which compares with (*Physics Letters*, loc. cit.) $K^{\pm} = 493.82 \pm 0.11$ and $K^0 = 497.76 \pm 0.16$. In the above result the behavior of the charges is obtained from (6.16), where

$$j_t = x_t h_t - (i/c)(\partial h_t/\partial t) = 2x_t h_t \qquad \text{or} \qquad 0$$

depending upon the direction of rotation.

16.5.6. *The ψ particles.* (Reported in *New Scientist*, 19 December 1974, pp. 870–872.) The announcement of the first ψ particle led the writer to try a modification of the kaon formula, i.e., assume an electron and positron at the critical velocity $c/\sqrt{2}$. Such particles would exhibit unit charge as in the kaon. Hence we have

$$\psi = 2(5/\sqrt{2})m_e(md^2/I)^3$$

then

$$\psi = 3.62 \text{ GeV}$$

which compares with 3.695 GeV reported for the second ψ particle ψ (3695).

The other alternative seemed to be a critical velocity at which the magnetic charge of the Λ particle would be canceled. Thus the mass would be basically

$$\psi = \Lambda y_{3/2,1}$$

plus correcting terms because the particles are assumed to be moving along the magnetic axis instead of rotating. Thus the corrections are $2\alpha(1 - 1/4\pi^2)^{-1/2}$ for relativistic rotation and $2\alpha(1 - m_e/M_p)^{-3}$ for center of mass. This gives

$$(1115.51 - 5.1776 - 0.5736)2.798386$$
$$= 3.10554 \text{ GeV}$$

or the ψ (3105) particle. Based on these models, one would have expected the ψ (3695) to have been seen at Stanford and the ψ (3105) at Brookhaven since the former is working with a positron and electron colliding beam and Brookhaven with protons. However, Brookhaven were looking at electron and positron behavior after proton–proton collisions, and of course, provided that the energy is right, any of these particles are possible. Nevertheless, it is just possible that the protons in the Brookhaven experiment disintegrate owing to the cancellation of their magnetic charge, which would represent a new phenomenon.

17. *Gravitation*

Milner suggested that his h_t force might be connected with gravitation. If this can be shown to be true we shall have completed a unified field theory in which all the forces of nature are seen to be interdependent. Einstein was clearly aware of this, for he entitled a paper in 1919 "Do Gravitational Fields Play an Essential Part in the Structure of the Elementary Particles of Matter?" The answer below agrees with his conclusion "there are reasons for thinking that the elementary formations which go to make up the atom are held together by gravitational forces."

In the derivation of Eqs. (6.9)–(6.18) it was tacitly assumed that fields are continuous; i.e., charge densities, for example, are actual continuous densities, not the total charge, when made up of discrete charges, divided by the volume. The question therefore arises—What is the value of x_t in the regions between charges? Is it zero or just some very small value?

Now a little algebra will show that the condition that j_t should be zero requires $\nabla^2 h_t = 0$ but not that $\nabla^2 \mathbf{e}$ be zero. Similarly for k_t and $\nabla^2 \mathbf{h}$ and $\nabla^2 e_t$. It follows that in the wave equation for h_t

$$(1/c^2)(\partial^2 h_t/\partial t^2) = x_t^2 h_t \tag{17.1}$$

so that x_t need not be zero for the inverse square law to remain valid.

Rewriting (6.16) before resolutes are taken, we have

$$\left(\operatorname{div} \mathbf{e} - x_t h_t + \frac{1}{c}\frac{\partial e_t}{\partial t}\right) + i\left(\operatorname{div} \mathbf{h} - x_t e_t + \frac{1}{c}\frac{\partial h_t}{\partial t}\right) = 0 \tag{17.2a}$$

$$i\left(\frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} - \operatorname{curl} \mathbf{h} - x_t \mathbf{h} + \operatorname{grad} e_t\right) - \left(\frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} + \operatorname{curl} \mathbf{e} - x_t \mathbf{e} + \operatorname{grad} h_t\right) = 0 \tag{17.2b}$$

Then for div $\mathbf{e} = 0 = $ div $\mathbf{h}$, obvious solutions are

$$h_t = A \exp(\omega t) = A \exp(\omega t) \tag{17.3a}$$

$$h_t = A \exp(-i\omega t) = B \exp(i\omega t) \tag{17.3b}$$

so that

$$-x_t + \omega/c - ix_t + i\omega/c = 0$$

or

$$-x_t h_t + (\omega/c)h_t - ix_t e_t + i(\omega/c)e_t = 0$$

and $x_t = \omega/c$ in both cases. Equation (17.3a) is clearly too restricting, so we shall choose (17.3b), which with (17.1) suggests an oscillating universe, in which case x_t bears some relation to Einstein's cosmological constant (1917). Although he disclaimed the method as a means of removing the difficulties of Newtonion theory, Einstein (loc. cit.) wrote in place of Poisson's equation

$$\nabla^2 \phi - \lambda\phi = G\rho \tag{17.4}$$

where ϕ is the gravitational potential, λ is the cosmological constant, G is the gravitational constant, and ρ is the gravitational mass density.

Applying the assumption $\nabla^2\phi = 0$ gives

$$\lambda\phi = -G\rho$$

Comparing with the wave equation and remembering (17.3b) leads to the suggestion

$$x_t^2 = \lambda = 1/R^2 \tag{17.5}$$

where R is the radius of the spherical universe (Einstein, 1917). We shall now pursue this suggestion by considering Milner's equations for matter in motion.

Milner shows that the rate of change of momentum is given by

$$\frac{\partial}{\partial t}\frac{1}{2}\frac{1}{c}(e_t^2 + h_t^2) + \frac{\partial}{\partial x}(e_t e_x + h_t h_x) + \frac{\partial}{\partial y}(e_t e_y + h_t h_y) + \frac{\partial}{\partial z}(e_t e_z + h_t h_z)$$
$$= \mathbf{e}j_t - \mathbf{j} \times \mathbf{h} - \mathbf{h}k_t - \mathbf{k} \times \mathbf{e} \tag{17.6}$$

The first term represents the time rate of change of "t-momentum," and we may write the force equation as

$$\mathrm{F} = \frac{1}{c}\left[e_t \frac{\partial e_t}{\partial t} + h_t \frac{\partial h_t}{\partial t}\right] \tag{17.7}$$

Considering only the h_t terms and the condition in which a particle has zero velocity in a field h_{tf}, we may write, since they are scalar quantities, for the region inside the particle

$$h_t = h_{tp} + h_{tf} \tag{17.8}$$

where h_{tp} is the particle's internal field. From (17.8) and (17.3b) we have

$$\frac{\partial h_t}{\partial t} = -ix_t ch_{tf} \tag{17.9}$$

So the amplitude of the force at the sink particle due to the field from the source is

$$F_h = -ix_t ch_t h_{tf} \tag{17.10}$$

To simplify the algebra, we assume that h_t in both particles obeys (7.1) so that

$$h_{tf} = \frac{kA_p}{gl^2} \frac{\sin x_p r}{r} \tag{17.11}$$

where l is the separation between source and sink particles, m_s is the mass of the source particle, m_p is the mass of the sink particle, g is a dimensional constant = length^{-2}, $k = m_s/m_p$, and $l \gg \pi/x_p$. x_p is the value of x within either the source or sink particle which may or may not carry equal electric charge, so that masses will be assumed proportional to the A's. x_t remains the value of x "in vacuo."

To obtain the total force exerted on the sink, integrate all over the source, since it exerts a force at each point of the sink, i.e., integrate (17.11) and multiply by the value of $x_p h_t$ at the point in question. Since, however, the force is measured and defined as that which acts on unit mass, we must also integrate all over the sink and divide by its mass. Thus we obtain

$$\frac{F_T}{m_p} = 4\pi A_p A_f \frac{x_t}{l^2 g} \int_0^{r=\pi/x_p} \sin^2 (x_p r)\, dr \tag{17.12}$$

$$= \frac{4\pi A_p{}^2}{2x_p l^2 g} kx_t$$

$$= \frac{Wk}{2l^2 g} x_t \tag{17.13}$$

by using (7.4).

Writing $W = m_p c^2$ we have

$$F_T = \frac{m_p m_s}{l^2} \frac{x_t c^2}{2g} \tag{17.14}$$

But

$$F_T = G \frac{m_p m_s}{l^2} \tag{17.15}$$

where $G = 6.668 \times 10^{-8}$ g^{-1} cm^3 sec^{-2} (Allen, 1955). Comparing (17.14) and (17.15)

$$Gg = x_t c^2/2$$

So $2/x_t = 1.347 \times 10^{28}$ cm and a "half-wavelength" $1/2x_t = 3.37 \times 10^{27}$ cm, which compares with 3.2×10^{27} cm quoted by Allen as the radius of the observable universe.

18. *Experimental Checks*

At the time of writing a newspaper reported that Van Flandern, of the U.S. Naval Observatory, has shown that two inches of the Moon's annual recession from the Earth is unaccounted for by tidal friction and has suggested that the discrepancy may be explained by an annual decay of the gravitational constant of about one part in 10^{10}.

The above calculation compares with this:

$$x_t c \triangleq \frac{3 \times 10^{-8}}{0.6735} = 4.46 \times 10^{-8}\ \text{sec}^{-1}$$

$$= 1.4 \times 10^{-10}\ \text{yr}^{-1}$$

19. *Interpretation*

From Eqs. (6.16) it may be seen that the signature of h_t is related to that of the charge density. Further, from (6.16d) for a stationary particle, we see that both positive and negative h_t represent a tension. Thus we may conclude that like charges (of h_t) attract and unlike charges repel by cancellation, which is consistent with the choice of the solution with the negative exponential for both charges in Eq. (17.3b) and the concept of an expanding universe. A positive exponential for negative charges would lead to asymmetrical conditions in, for example, positronium. Since the proton is so much heavier than the electron, it is difficult to decide how to put the theory to an experimental test. Witteborn and Fairbank (1967, 1968), who discuss the whole question of the gravitational properties of elementary particles, found that the force on an electron was less than 0.09 mg, where m is its inertial mass and g is 980 cm/sec^2. They incline to the belief that their experiment supports the contention that "gravity induces an electric field outside a metal surface, of magnitude and direction such that the gravitational force on electrons is cancelled." (Schiff & Barnhill, 1966.) However, their proposed experiment with positrons might conceivably avoid this difficulty.

So far in the analysis e_t has been neglected on the grounds that it is dipolar in nature and its effect is therefore small. However, its dipolar nature seems to render the first exponential solution of (17.2) unlikely, and the terms become oscillatory. If we accept this second solution then e_t and h_t are independent, and it is also of interest that e_t is then an antigravitational force and compressive. Thus we may visualize e_t forces in the proton holding the magnetic charges apart. The possibility, therefore, that ψ (3105) may be generated by a cancellation of the magnetic charge within the proton would suggest a possible first step in the "release" of antigravitational forces and the observation of the magnetic monopole.

20. *Conclusion*

It has been shown how Milner's theory, based upon the factorization of matter into electric and magnetic quaternions, may be extended to include the force on a moving conductor; quantization allows models of the proton and electron to be constructed which exhibit the correct parameters including magnetic moments, and from these two it appears that all other particles may be constructed.

To illustrate this the paper includes models of the n, Λ, Σ^+, and K particles and tentative models of the two recently discovered ψ particles. I have also built models of π and μ mesons and have made some headway with the masses of the Ξ and Ω^- baryons; but as I am not satisfied that I yet fully understand the method of adding charges to the basic particles (e.g., to obtain Σ^-), and as this part of the work is not necessary to the main theme of the paper, I have thought fit to omit the calculations. Similar considerations have prompted omission of mention of the neutrino.

Finally an attempt has been made to show that Einstein and Milner's belief, that the cohesive forces of the proton and electron are the source of gravitational forces, is at least plausible.

Apart from the aspects of the theory which lead to calculable data suitable for experimental verification, I have attempted throughout to show that the new theory subsumes relativity and quantum theories rather than conflicting with them and that the conflict, which has been supposed to exist hitherto, is an illusion generated by metaphysical speculation and induction beyond the facts.

It seems appropriate to end with Hamilton's own words in the preface of the paper which has made all this theory possible, for it so expounds my own position that to reword it for my own use would be plagiarism.

> And if for the present I omit all further mention of them, it is partly because, without a closer study, I should fear to do them an injustice; and partly because I make no pretensions to be here an historian of science, even in one department of mathematical speculation, or to give more than an account of the progress of my own thoughts, upon one class of subjects.

Appendix I: The ψ Particles

Shortly after the announcement of the discovery of ψ (3695) and of ψ (3105) came that of a new neutral particle ψ (4.1). The mass of such a particle may be arrived at by taking ψ (3105) and, as for Σ^+, multiplying the mass of Λ by $j_{3/2,1}/\pi$ and subtracting the mass of the "electron" in that particle, i.e.,

$$\psi\,(4.1) = (4.49/\pi)\,(3100 - 175.3) = 4.179 \text{ GeV}$$

Such a particle will have zero charge and max h_t at the boundary.

The importance of these particles lies in the possibility of the experiment pointing the way to a release of e_t forces and magnetic charge; otherwise in themselves they seem to be of no great importance. Of course, to release e_t forces one would need to split the proton and not just come into resonance with its fields with an electron or positron as seems to have been happening.

Appendix II: The "Stable" Particles

Models of Ξ^0 Ξ^- and Ω^- have now been constructed which give support to the model of the proton and show an intelligible pattern for the "stable" particles with masses between p and Ω^- inclusive. The constituent particles are the proton, the orbiting particle as in the Λ particle [Eq. (16.12)], and the oscillating particle as in the neutron. The last two will be called field electron rotating (f.e.r.) and field electron oscillating (f.e.o.), respectively. Both the proton and the field electron may exist with zero charge and maximum h_t at the boundary. Such charges are termed neutral. The neutral proton may have its magnetic charge either unexcited as in the charged proton or in the next excited state.

The models are then

		Masses, MeV Calculated	Experimental
n	$1p + 1$ f.e.o.	$1.29356 + M_p$	$1.2935 + M_p$ ± 0.00001
Λ	$1p + 1$ f.e.r.	1115.51	1115.6 ± 0.08
Σ^+	$1p + 1$ neutral f.e.r.	1189.4078	1189.40 ± 0.19
Ξ^0	1 neutral p with magnetic charge unexcited	1314.76	1314.7 ± 0.7
Ξ^-	1 neutral p with magnetic charge unexcited + neutral f.e.o. coupling through h_t forces with neutral p	$\Xi^0 + 6.5$	$\Xi^0 + 6.6$ ± 0.7
Ω^-	1 neutral p with magnetic charge in first excited state + neutral f.e.r. {coupling as for Ξ^-}	1672.416	1672 ± 0.5

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References

Allen, C. W. (1955). *Astrophysical Quantities.* Athlone Press, London.
Berkson, W. (1974). *Fields of Force.* Routledge and Keegan Paul, London, ISBN 0 7100 7626 6.
Born, Max (1951). *Atomic Physics,* 5th ed. Blackie, London.
Born, Max (1969). *Physics in my Generation.* Longman and Springer Verlag, London.
Born, Max (1971). *Born–Einstein Letters.* Macmillan, New York.

Cayley, Arthur (1895). *Elliptic Functions.* Chap. III. George Bell; 2nd ed., Dover Publications, New York.

Chiu, H-Y. and Hoffmann, W. F. eds. (1964). *Gravitation and Relativity,* W. H. Benjamin, New York.

Cohen, E. R. and du Mond, J. W. M. (1957). "The Fundamental Constants of Atomic Physics," *Handbuch der Physik,* **35**, 43.

Corben, H. C. and Stehle, P. (1950). *Classical Mechanics.* John Wiley & Sons, New York.

Dicke, G. H. (1957 & 1963). *American Institute of Physics Handbooks,* American Institute of Physics, New York.

Dirac, P. A. M. (1948). *Physical Review,* **74**, 817.

Einstein, A. (1917). "Cosmological Considerations of the General Theory of Relativity." translated from the German by W. Perrett and G. B. Jeffrey in *The Principle of Relativity,* p. 175. Dover Publications, New York.

Einstein, A. (1919). "Do Gravitational Fields Play an Essential Part in the Structure of the Elementary Particles of Matter?" translated from the German by W. Perrett and G. B. Jeffrey in *The Principle of Relativity,* p. 189. Dover Publications, New York.

Elsasser, W. M. (1973). *Foundations of Physics,* **3**, No. 1, pp. 117–137.

Fermi, E. (1951). *Elementary Particles.* Oxford University Press, London.

Gabor, D. (1960). "A note on Samuel Roslington Milner 1875–1958," *Biographical Memoirs of Fellows of the Royal Society,* **5**, 129. Quoted by Kilmister in *Generalised Electrodynamics and the Structure of Matter* (see S. R. Milner).

Harvey, I. K., Macfarlane, J. C. and Frenkel, R. B. (1970). *Physical Review Letters,* **25**, 853.

Hamilton, W. R. (1967). *The Mathematical Papers of Sir William Roman Hamilton,* Vol. VI, *Algebra,* Alberstain, H. and Ingram, R. E., eds. Cambridge University Press, Cambridge.

Heitler, W. (1963). *Man and Science.* Oliver and Boyd, London.

Jackson, J. D. (1962). *Classical Electrodynamics,* Wiley, New York (quoted by Berkson) pp. 338–9.

Jeffreys, H. and Jeffreys, B. S. (1950). *Mathematical Physics,* 2nd ed., p. 320. Cambridge University Press, Cambridge.

Joos, G. (1958). *Theoretical Physics* 3rd ed. Blackie and Son, London.

Kilmister, C. W. (1971). "Beyond What?" in *Quantum Theory and Beyond,* Ted Bastin, ed. Cambridge University Press, Cambridge.

Lai, S. B., Knight, P. L. and Eberly (1974). "Electron's Anomalous Moment and its Spin-Precession Frequency Shift," *Physical Review Letters,* **32**, 494.

Matts Roos et al. (1969). *Physics Letters,* **33B**, 4.

McLachlan, N. W. (1950). *Ordinary Non-Linear Differential Equations in Engineering and Physical Sciences.* Oxford University Press, Oxford.

Milner, S. R. (1960). Philosophical Transactions of the Royal Society of London, **S253**, 185, 205.

Milner, S. R. (1963). *Generalised Electrodynamics and the Structure of Matter,* C. W. Kilminster, ed. University of Sheffield, Sheffield.

Milne-Thomson, L. M. (1950). *Jacobian Elliptic Function Tables.* Dover Publications, New York.

Parker, W. H., Taylor, B. N. and Langenberg, D. N., *Physical Review Letters,* **18**, 287.

Piaget, J. (1970). *Genetic Epistemology.* Columbia University Press, New York.

Picot, A. (1961). *On Modern Physics. W. Heisenberg, M. Born, E. Schrodinger and P. Auger,* p. 21. The Orion Press, London.

Popper, K. R. (1972). *Objective Knowledge.* Oxford University Press, London.

Post, H. R. (1971). "Correspondence, Invariance and Heuristics: In Praise of Conservative Induction." *Studies in the History and Philosophy of Science,* **2**, No. 3.

Russell, B. (1946). *History of Western Philosophy,* pp. 832 and 239. George Allen and Unwin, London.

Samual, Viscount (1951). *Essay in Physics.* Basil Blackwell, London (contains a letter from Einstein pp. 136–145, quotation pp. 143–145).

Schiff, L. I. and Barnhill, M. V. (1966). *Physical Review*, **151**, 1067.
Scheibe, E. (1973). *The Logical Analysis of Quantum Mechanics.* Pergamon Press, London, ISBN 0-08-017158-3.
Sommerfeld, A. (1934). *Atomic Structure and Spectral Lines,* 3rd ed., Vol. 1. Methuen, London.
Stratton, J. A. (1941). *Electromagnetic Theory,* pp. 438 ff. and pp. 488 ff. McGraw-Hill, New York.
Synge, J. L. (1972). *Quaternions, Lorentz Transformations, and the Conway-Dirac-Eddington Matrices,* pp. 1–50. Dublin Institute for Advanced Studies, Dublin.
Thom, R. (1969). *Topology,* **8**, 313.
Thomson, J. J. (1921). *Elements of the Mathematical Theory of Electricity and Magnetism,* 5th ed., Chap. XV. Cambridge University Press, Cambridge.
Toulmin, S. (1967). *Synthese*, **17**, 75.
White, H. E. (1931). *Physical Review*, **37**, (1416).
Witteborn, F. C. and Fairbank, W. M. (1967). *Physical Review Letters,* **19**, 1049.
Witteborn, F. C. and Fairbank, W. M. (1968). *Nature,* **2**, 436.